VP160 Recitation Class

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Kinematics in 3D: Cartesian Coordinates

Basic Formulas

$$\vec{r} = x(t)\hat{n}_x + y(t)\hat{n}_y + z(t)\hat{n}_z$$
$$\vec{v} = \dot{x}(t)\hat{n}_x + \dot{y}(t)\hat{n}_y + \dot{z}(t)\hat{n}_z$$
$$\vec{a} = \ddot{x}(t)\hat{n}_x + \ddot{y}(t)\hat{n}_y + \ddot{z}(t)\hat{n}_z$$

Kinematics in 3D: Cylindrical and Spherical Coordinates

Basic Formulas

$$\vec{r} = \rho\hat{n}_\rho + z\hat{n}_z$$
$$\vec{v} = \dot{\rho}\hat{n}_\rho + \rho\dot{\phi}\hat{n}_\phi + \dot{z}\hat{n}_z$$
$$\vec{a} = (\ddot{\rho} - \rho\dot{\phi}^2)\hat{n}_\rho + (\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi})\hat{n}_\phi + \ddot{z}\hat{n}_z$$

Remind: Do remember these formulas, otherwise you may derive them by yourselves during the exam!

Kinematics in 3D: Natural Coordinates

Basic Vectors

1. $\hat{n}_\tau$: along the direction of $\vec{v}$
2. $\hat{n}_n$ and $\hat{n}_b$: perpendicular to the direction of $\vec{v}$

Kinematics in 3D: Natural Coordinates

Basic Vectors

1. $\hat{n}_\tau$: along the direction of $\vec{v}$
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Basic Formulas

$$\hat{n}_\tau = \frac{\vec{v}}{\|\vec{v}\|}$$

$$\hat{n}_n = \frac{d\hat{n}_\tau/dt}{\|d\hat{n}_\tau/dt\|}$$

$$\hat{n}_b = \hat{n}_\tau \times \hat{n}_n$$

$$\vec{v} = v\hat{n}_\tau$$

$$\vec{a} = \dot{v}\hat{n}_\tau + \frac{v^2}{R_c}\hat{n}_n$$

Important Tips

1. Differences between $\dot{\vec{v}}$ and $\dot{v}$.
2. Differences between radial/transversal, normal/tangential.

Kinematics in 2D: Polar Coordinates

Basic Formulas

$$d\vec{r} = d\rho\hat{n}_\rho + \rho d\phi\hat{n}_\phi$$
$$(ds)^2 = (d\rho)^2 + (\rho d\phi)^2$$

Exercise 1

A particle moves in the x-y plane so that

$$x(t) = at,\ y(t) = bt^2$$

where a, b are positive constants. Find its trajectory, velocity, and acceleration (its tangential and normal components).

Exercise2

A disc of radius R rotates about its axis of symmetry (perpendicular to the disk surface) with constant angular velocity $\dot{\phi} = \omega =$ const. At the instant of time $t = 0$ a beetle starts to walk with constant speed v_0 along a radius of the disk, from its center to the edge. Find:

(a) the position of the beetle and its trajectory in the Cartesian and polar coordinate systems

(b) its velocity in both systems,

(c) its acceleration in both systems,

(d) the distance covered by the beetle and the curvature of the trajectory.

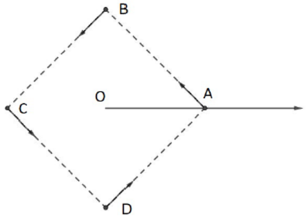
Exercise 3

Four spiders are initially placed at the four corners of a square with side length a. The spiders crawl counter-clockwise at the same speed v and each spider crawls directly toward the next spider at all times. They approach the center of the square along spiral paths. Find:

(a) polar coordinates of spider A at any instant of time *t*

(b) the time after which all spiders meet

(c) the trajectory of a spider in polar coordinates



Exercise 4

A particle moves along a hyperbolic spiral (i.e. a curve $r = c/\phi$, where c is a positive constant), so that $\phi(t) = \phi_0 + \omega t$; where ϕ_0 and ω are positive constants. Find its velocity and acceleration (all components and magnitudes of both vectors).